**Exponential and Logarithmic Graphical Behavior Lesson Activity**

**Mathematical Learning Objectives:**

Determine the graphical behavior from varying parameters a, h, k and b of the exponential form y=a\*(b)x-h+k

Identify domain, range, asymptotes and intercepts of exponential functions.

Understand the transformation of an exponential parent function when varying multiple parameters.

**Language Objective:**

Students will be engaged in this lesson activity which will allow students to explore and view multiple representations of exponential forms of functions and reason quantitatively to explain the graphical behaviors observed. Students will be focused on describing the behaviors of the graph when varying parameters, identifying the domain, range, and intercepts of exponential functions and observe how the behaviors differs when multiple parameters are changed. Students will have an opportunity to relate graphical behaviors to equivalent forms of exponential equations and see how this compares to arithmetic procedures.

**Essential Question:**

What is the graphical behavior when varying parameters of an exponential function?

**Common Core State Mathematics Standards:**

F-IF.7.e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

**Common Core State Mathematical Practice Standards:**

Make sense of Problems and Persevere in Problem Solving, Reason Abstractly and Quantitatively, Look for and Express Regularity in Repeated Reasoning, and Look for and Make Use of Structure.

**Materials:**

Calculator, paper, pencils, computers and Geogebra Software.

**Notes to the Reader:**

Students must understand computing arithmetic, applying properties of exponential functions. Students should know the parent function of exponential forms or be introduced to this before the lesson activity. Students must know how to solve for intercepts of functions. Students must know how to describe domain and range using interval notation.

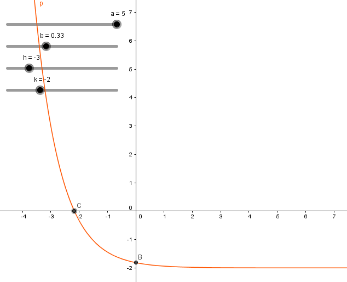
Dave- you are off to a really good start. I would like for you to develop a student handout that clearly outlines the problems students are working on and the questions you want them to answer. What is confusing is the role of the GeoGebra file. You have two different tasks here. Your lesson plan instructs students to graph certain functions and to describe the graphs. The GeoGebra file has students vary the parameters, and I assume to notice what happens with each changing parameter. They are both good activities, but decide which you want to use. It seems like you want students to examine each parameter one at a time and describe the graph starting with y=2x. Then, students vary b, then vary a, then vary h and finally, vary k. Consider students looking at a variety of functions within each parameter- so we start with the parent function, then we look at 3 or 4 functions where b is changing (perhaps y=3x, y=4x, y=(1/2)x, y=(1/3)x. Then, ask “What happens to the graph when b>0 ? What happens to the graph when 0<b<1?” Then, do the same for a, h, and k. The idea is to get students to see patterns when they change one parameter.

Then, perhaps a culminating activity can be where they confirm their observations using the GeoGebra file. Let’s meet to iron out these details.

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| **Time (min)** | **Teacher Actions** | **Student Engagement** |
| **10** | **Introduction/Review**  “Today we are going to be doing a lesson activity exploring the graphical behavior of exponential functions.”  “Can someone recall the form of an exponential function?”  “You are correct, but today we are going to investigate what happens when we vary the parameters of the exponential function in the form  y=a\*(b)x-h+k where a≠0, b>0 and b≠1.”  “So what happens to the function if a=0?  b=1? b=0? When b is negative?”  “Throughout the lesson activity you will list the domain, range and y intercepts, make observations of graphical behaviors when varying each parameter. There are also other items I ask you to identify in problem 1 which can help with descriptions of the graphical behaviors on the rest of the activity. Let go over these items.”  “What is an asymptote?”  “How can one describe end behaviors of a linear function? How does this relate to asymptotes?”  “Can anyone recall interval form of how to write domain and range values?”  What is the domain y=x2  What is the range of y=x2 and why? | “The form of an exponential function is y=(b)x.”  ***Mention the following If not answered.***  “If a=0 or b=0 then the function is k for all x values. This is a line and we are not interested in interpreting graphical behaviors of lines. If b=1 then y=a+k because bx will equal 1. Again this is a line and we are not interested in interpreting graphical behaviors of lines. If b is negative, then the function oscillates between negative and positive values and is discontinuous at even fractional roots.  “An asymptote is a vertical line on a graph that represents an input value that is undefined on the function.”  **If not answered, then explain the following:**  “End behaviors for a linear function can be described as x approaches infinity then y increases to infinity and as x approaches negative infinity then y decreases to negative infinity. We call this an increasing function because it increases from left to right. This is also a function that is continuous because there are no undefined x or y values in this function and the graph shows this. This relates to asymptotes because we will need to describe end behaviors that approach asymptotes.”  “The domain of y=x2 is D:(-∞,∞)  And R:(0,∞) this is because a positive square function will always have positive outputs and all x values are defined.” |
| **50** | **Lesson Activity**    Figure 1, Problem 1: y=2x    Figure 2, Problem 4: y=(1/3)x  **Other Teacher Facilitation Questions to Problems 1-4:**  Does the y intercept always equal 1?  Is the graph touching the x-axis? Explain how you determine this.  Is it always true for exponential functions that the graph only contains positive y values?  Does the shape of the graph change? What happens? Why?  **Other Teacher Facilitation Questions, Problems 5-8:**  How did you determine the graph is moving to the right 2 units?  Can you show me how you mathematically justify this?    Figure 3, Problem 5: y=2(x-2)    Figure 4, Problem 7: y=2(x+3)    Figure 5, Problem 9: y=2x-1    Figure 6, Problem 11: y=2x+3  **Other Teacher Facilitation Questions, Problems 9-11:**      Figure 7, Problem 13: y=2\*2x    Figure 8, Problem 14, y=-3\*2x  **Other Teacher Facilitation Questions, Problems 12-15:** | ***Part 1)***  ***Graph the following in your calculator and then answer the conclusion questions at the bottom of each section. Record the domain, range, asymptotes and intercepts for each problem.***  **Problem 1**  Graph the function: y=2x  **List the following:**  Y-intercept: (0,1)  Domain: (-∞,∞)  Range: (0,∞)  Asymptote: y=0  **Problem 4**  Graph the function: y=(1/3)x  **List the following:**  Y-intercept: (0,1)  Domain: (-∞,∞)  Range: (0,∞)  Asymptote: y=0  **Student Response to other facilitation questions:**  “The y intercept always equals one for these problems because anything to the power of 0 is always 1.”  “No the graph doesn’t touch the x axis. The function is undefined at y=0. There is no value x when 0=2x. Yes, there are no negative outputs for an exponential function of the form y=bx.  “Yes, when b is less than 1 the graph flips over the y axis. This happens because as x increases the output decreases, but still cannot have a negative output.”  **Answer the following conclusion questions (Req’d):**  **What happens when b>1?**  “When b is greater than 1 the graph is an increasing function and the greater the value of b the more compressed the graph becomes. When x>0 the graph becomes more steep with greater b values. As x decreases from zero to about x=-1, the graph decreases towards the asymptote y=0 at a faster rate with greater b values. Increasing b from 1 gives a steeper slope of the graph.”  **What happens when 1>b>0?**  “Similarly, when b decreases from 1 this results in a steeper slope of the graph. As x decreases from 0 the y values are greater for smaller b values. As x increases from 0 to 1 the y values are less for smaller b values. When b is between 0 and 1 the graph reflects around the y axis and y intercept is unchanged.”  **What other observations did you make while changing b?**  “The function doesn’t act as a typical exponential when b is less than or equal to zero. When b is 1 the function is linear.”  **Problem 5**  Graph the function y=2(x-2)  **List the following:**  Domain: (-∞,∞)  Range: (0,∞)  Y intercept: (0,1/4)  Asymptote: at y=0  **Problem 7**  Graph the function y=2(x+3)  **List the following:**  Domain: (-∞,∞)  Range: (0,∞)  Y intercept: (0,8)  **Student Response:**  “I determined this by observing the y-intercept of (0,1) from the graph y=2x moved to (2,1) when graphing y=2(x-2).  Any number raised to the power of zero is one. I set both exponents equal to zero (x-2=0, x=0) and solved for x. So y=2x and y=2x-h both equal 1.”  **Answer the following conclusion questions (Req’d):**  What happens when h is negative?  “When h is negative the graph shifts to the left h units.”  What happens when h is positive?  “When h is positive the graph shifts to the right h units.”  What other observations did you make while changing h?  “When h is negative the y intercept is less than 1 and when h is positive the y intercept is greater than 1. The domain and range are the same when h is negative or positive the graph remained an increasing function.”    **Problem 9**  Graph the function y=2x-1  **List the following:**  Domain: (-∞,∞)  Range: (0,∞)  Y intercept: (0,0)  Asymptote: at y=-1  **Problem 11**  Graph the function y=2x+3  **List the following:**  Domain: (-∞,∞)  Range: (0,∞)  Y intercept: (0,3)  Asymptote at y=3  **Student Response:**  **Answer the following conclusion questions (Req’d):**  What happens when k is positive?  “When k is positive the graph shifts vertically up k units.”  What happens when k is negative?  “When k is negative the graph shifts vertically down k units.”  What other observations did you make while changing k?  “The y intercept is at the origin when k is -1. When k is less than -1 the y intercept is a negative output and when k is greater than -1 the y intercept is a positive output.”  **Problem 13**  Graph the function y=2\*2x  Domain: (-∞,∞)  Range: (0,∞)  Y intercept: (0,2)  Asymptote: at y=0  **Problem 14**  Graph the function y=-3\*2x  Domain: (-∞,∞)  Range: (0,-∞)  Y intercept: (0,-3)  Asymptote: at y=0  **Answer the following conclusion questions (Req’d):**  What happens when a is positive?  When a is positive the graph is increasing and opens upwards.  What happens when a is negative?  When a is negative the graph is decreasing and reflects around the x axis.  What other observations did you make while changing a?  The y intercept was always equal to a for these problems. |
| 20 | **Whole Class Discussion**  **Have students explain their findings from varying parameters of the exponential function.**  **Make sure to select the groups that have different solution strategies that explain their observations. This could be equivalent expressions, table of values, or concentrating on output points such as the y intercept.**    **Student Demonstrations of Work:**  “Group A has demonstrated their abilities to quickly seeing the graphical behavior when varying k. Can you please explain your work.”  “Group B has done a good job at demonstrating their explanation of why the graph shifts horizontally. Can you please explain your thinking?”  “Group C has demonstrated an equivalent form. They have transposed y=a\*(b)x-h+k to y=a\*(bx\*b-h)+k so why is this important?”  **Evidence of Student Engagement:**  “In the group sharing we didn’t talk about varying a. What behavior can be seen by varying a? Why?”  “Does the domain and range stay the same when varying a? Why?”  **Teacher Wrap Up/Explanation for HW:**  “When the rate is increasing we refer to this behavior as a stretch and when decreased we refer to this as a compression.”  “For the homework recall that log(x) is the inverse function of 10x. Sometimes it is helpful to solve for intercept by applying the inverse function. Determine the inverse to locate the intercepts.” **Write example on board:**  y=-3x-2+2. Set y=0: 0=-3x\*3-2+2  so -2=3x\*(-1/9) then 18=3x apply inverse: log(18)=log(3x) so log(18)=x\*log(3) then log(18)/log(3)=x=2.6 | **Student Demonstrations of Work:**  **Group A:**  “We focused on one point at first and then saw that each point is shifted vertically. So first we have y=2x and when x=0 this is the y intercept value. If we compute this value for any value k that is added to this, then the output is increased by k units.”  **Group B:**  “We first made a table of values to verify the input and output values for a few points. We noticed the graph shifted in the x direction by h units. For example if b=2, h=-3 and x=2 then bx-h=2(2-(-3))=25=32  A phase shift to the right would come from decreasing h by one unit because x is subtracted by h. If y remains the same and the shift is one unit to the right where x=3 then h=-2. We showed this by solving x-h=5 where x=3. Since the base stayed the same, the exponent must remain the same to get the same output.”  **Group C:**  “We first put y=a\*(b)x-h+k into the form y=a\*(bx\*b-h)+k to see how h and x are related since we noticed the graph shifted horizontally with varying h we then concluded that since they share the same base that both shared a common factor of b. When h is increased by one unit then b-h loses a factor of b so x must increase by a factor of b to yield the same output. So we show this through multiplication of the factors. For example when b=2, a=1, h=-1 and x=2 we have bx\*b-h=(2\*2)\*(2)=8. If h was decreased by 1 unit where h=-2 then x will shift to the left to have the same number of 2’s. So we have bx\*b-h=(2)\*(2\*2)=8 where x=1.  **Evidence of Student Engagement:**  “The rate grows a times the exponential function. If a were less than one but greater than zero this would decrease the rate of growth because the output from the exponential function would be multiplied by a number less than one. If a were greater than one this would increase the rate of growth and the graph would stretch because all of the output values from the exponential function would be multiplied by a number greater than one. If a were negative than this would flip the graph over the horizontal asymptote because all of the output values would be opposite the exponential function.”  “Yes, the domain always remains the same when varying a but the range would change when a was negative.” |
| 10 | **Extension**  **Graph these problems for a homework assignment and identify the domain, range, asymptotes, end behavior, intercepts and if the graph is increasing or decreasing.** | **Homework Problems**   1. Y=5(1/3)x+3-2 2. Y=-1/2(2)x-1+4 |

**Homework Solutions:**

Problem 1: y=5(1/3)x+3-2

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Domain: (0,∞)

Range: (-∞,-2)

End Behavior: As x approaches infinity y approaches -2 because 5(1/3)x+3 can only be positive values and then the graph gets shifted down k units. So the asymptote moves from y=0 to y=-2. As x approaches negative infinity y approaches infinity.

Asymptote at y=-2

The graph is decreasing because b is less one but greater than zero. The outputs will be smaller when x increases because when a fraction is multiplied to another fraction this reduces the value.

x-intercept:

Graphical find x=-2.17

Or solve by applying the inverse function, log() / set y=0

0=5\*(1/3)x\*(1/3)3-2

2=(5/27)\*(1/3)x

((27\*2)/5)=(1/3)x

(Log(27)+log(2))-log(5)=log((1/3)x)

((Log(27)+log(2))-log(5))=xlog(1/3)

((Log(27)+log(2))-log(5))/(log(1/3))

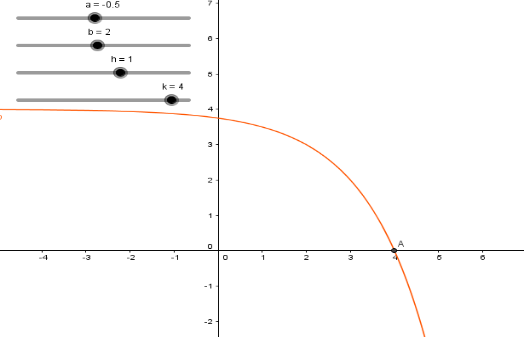
x=-2.17

y intercept: Graphical find y=-1.81

Solve by setting x=0:

y=5\*(1/3)0\*(1/3)3-2=(5/27)-2=(5-54)/(27)=-1.81

Problem 2: y=-1/2(2)x-1+4



Domain: (-∞,∞)

Range:(4,-∞)

End Behavior: As x approaches infinity y approaches negative infinity. As x approaches negative infinity y approaches 4.

The graph is decreasing because a is negative so the graph reflects over y=0 but then shifts up 4 units because k=4. Since a is greater than -1 and less than 0 the graph is compressed.

Asymptote at y=4

x intercept:

Graphically view x=4

Solve by setting y=0:

(-1/2)\*(2)x\*(2)-1+4=0

-4=(-1/2)\*(1/2)\*(2x)

-4=(-1/4)\*(2x)

-4\*-4=(2x)

Log(16)=log(2x)

X=log(16)/log(2)

X=4

Y intercept: Graphically find y=3.75

Set x=0:

Y=(-1/2)\*(2)0\*(2)-1+4=(-1/2)\*(1)\*(1/2)+4=(-1/4)+4=15/4=3.75